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A Simple Test for Normality *

by

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Abstract

The mean and the variance of a random sample are independently distributed if and only if the parent population is normal. This characterization is used as a basis for developing a test termed Z-test for the composite hypothesis of normality. The simplicity of Z-test results from the computational ease and the clean form of the finite sample null distribution of the test statistic Z. It compares reasonably in power with several well known tests of normality and is particularly suitable for a joint assessment of normality. A routine for computing the statistic and its P-value is given.

Key Words: Test for Normality; Characterizations of Normality; Independence; Correlation Coefficient; Joint Assessment of Normality.

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1. INTRODUCTION

The problem of testing a composite hypothesis of normality (mean and/or variance unknown) is perhaps the most common goodness-of-fit problem encountered in the current statistical practice. For example, an examination of residuals in order to ascertain the normality of the error variable is a routine part of regression analyses and the analyses of linear models in general. The tests proposed by Fisher (1932), Pearson (1930), Kolmogorov and Smirnov, and Cramér and von Mises are among the earliest and those due to Shapiro and Wilk (1965) d'Agostino (1971), and Filliben (1975) are among the latest procedures constructed especially for this problem. Moreover, well known EDF tests such as Kolmogorov and Smirnov, Cramer and von Mises, and Anderson and Darling, tests originally proposed for the goodness-of-fit problem in canonical form have now been variously adapted for testing composite hypotheses of normality (see Stephens (1974) and Green and Hegazy (1976)). A recent power study of such EDF tests (Green and Hegazy (1976)) views them as "the most powerful --- except that d'Agostino's and the less convenient Shapiro and Wilk tests of normality are comparable in power to these". A more recent test due to Vasicek (1976), based upon the entropy characterization of normality appears to compare reasonably with these in power and convenience, in that it requires no estimation of parameters, transformations to uniformity as in EDF tests or use of tables of coefficients as in the Shapiro and Wilk test.

In this paper we propose and study a test for the composite hypotheses of normality based upon the well known fact that the population is normal if and only if the sample mean and sample variance are independently distributed. This test, which is easy to motivate, is also simple to implement, for it does not

require ordering and transformations of observations, use of tables of coefficients or estimation of parameters. Moreover, unlike the null distributions of most other test statistics, which are generally available either in asymptotic form or in terms of a few Monte Carlo percentiles, the null distribution associated with the present test is shown to be very near normal in small samples. This permits an easy computation of the P-value of the test and consequently makes the joint assessment of normality simple when several independent samples are available. In Section 2, the test and a development of the null distribution of its statistic are presented. In Section 3, the power of the test is compared with the power of some well known tests. An example illustrating the joint assessment of normality is discussed in Section 4. Section 5 contains remarks on a number of related points. Fortran and APL routines which give P-values of the test are given in the appendix.

2. THE TEST FOR NORMALITY

2.1 The test statistic

Consider the problem of testing the composite hypothesis that a sample $X_1, X_2, \ldots, X_n$ of size n is from a normal population. It is well known, e.g., see Cramér (1946), and Kagan, Linnik and Rao (1973), that the hypothesis is true if and only if the sample mean $\overline{X}$ and the sample variance S^2 are independently distributed. Thus a test for the independence of $\overline{X}$ and S^2 is also a goodness-of-fit test for the normality. The apparent limitation that we have only one $(\overline{X}, S^2)$ for testing the independence of the pair can be circumvented in many ways. The most convenient of these is to obtain n means and the corresponding variances, from the n samples obtained by deleting one observation at a time. Even though the n pairs so obtained are not indepen-

dent, they can be used to estimate the extent of the dependence between the mean and the variance of samples from the population. Because of its simplicity, we wish to use the product moment correlation coefficient as a measure of this dependence.

Now the test based upon a product moment correlation coefficient is appropriate for testing independence in a bivariate normal distribution. However, one of the marginal distributions in the present case, the distribution of variance, is nonnormal even if the parent population is normal. This can be remedied to a considerable extent by applying the famous Wilson and Hilferty (1931) cube root transformation to the n variances in order approximately to normalize them. Because of the well known invariance of the correlation coefficient with respect to changes in scale and origin,

$$r = \frac{\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i / n}{\sqrt{\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2/n}\ \sqrt{\sum_{i=1}^{n} Y_i - (\Sigma Y_i)^2/n}}, \tag{1}$$

where

$$Y_i = [\sum_{j \neq i} X_j^2 - (\sum_{j \neq i} X_j)^2/n-1]^{1/3}, \quad i = 1,2,\ldots,n$$

equals in magnitude the correlation coefficient between $U_i = \sum_{j \neq i} X_j = \sum_{j=1}^{n} X_j - X_i$ and Y_i, which is the same as the correlation coefficient between the n means and the cube roots of n variance.

Instead of the correlation coefficient we propose a monotone function, the Fisher transform,

$$z = \frac{1}{2} \log \frac{1+r}{1-r} \tag{2}$$

of the r, given by (1) above, as the statistic for testing normality of the population, and reject the null hypothesis if

$$|Z| \geq \text{constant} , \tag{3}$$

where the constant is determined by the level of significance.

2.2 The null-distribution of Z

The above choice of the test statistic was based upon the expectation that the robust symmetry and even near normality of the distribution of Z (Cramér, p. 399) computed from n independent observations from bivariate distributions with symmetric marginals will hold in the present case, in which the bivariate population has symmetric or almost symmetric marginals but the observations are not independent. The results of the Monte Carlo experiment described below amply justify this expectation.

In order to estimate the null distribution of Z in small samples, 5 sets of 1000 samples of size n each, for 25 values of n ($n = 5,7,\ldots,100$), from a normal population, were simulated using Marsaglia's (1972) random number package. For each of the samples, Z as defined by (2) was computed, and thus 5 sets of 1000 Z's for each n were compiled. These were then used to construct 5 independent estimates each of the mean, variance, skewness, and kurtosis of the distribution of Z by computing the corresponding statistics for each set. Graphical representations of these estimates appear in Figures 1, 2 and 3. It may be observed that for every n, the skewness of the distribution of Z is negligible and its kurtosis very near normal. (Note that the excess of kurtosis of the logistic distribution which is regarded as very close to normal is 1.2 (Johnson and Kotz(1970), p. 6)). On the basis of this empirical but substantial evidence, the null distribution of Z is concluded to be symmetric

and very near normal with mean 0. To estimate the variance $\sigma_n^2(Z)$ of the distribution, a weighted regression analysis was performed for the polynomial regression of $[\sigma_n(Z)]^{-1}$ on n. The analysis yields

$$\sigma_n = \hat{\sigma}_n(Z) = (0.591730 + 0.143559\,n - 0.002235\,n^2 + 0.000016\,n^3)^{-1} \tag{4}$$

A similar analysis of the variation of the excess of kurtosis, $\gamma_{2,n}(Z) = \beta_{2,n}(Z) - 3$, with respect to $1/n$ gave

$$\gamma_{2,n} = \hat{\gamma}_{2,n}(Z) = -11.697157/n + 55.059097/n^2 \tag{5}$$

For most practical purposes, the null distribution of Z may be taken to be normal distribution with mean zero and variance σ_n^2 given by (4). If greater accuracy is desired, then the Edgeworth formula may be used to obtain

$$\Pr(|Z| \geq C) = P(|\frac{Z}{\sigma_n}| \geq C/\sigma_n) = 2 - 2\,P\{\frac{Z}{\sigma_n} \leq C/\sigma_n\}$$

$$\doteq 2 - 2\{\Phi(C/\sigma_n) - \frac{1}{24}\gamma_{2,n}[(C/\sigma_n)^3 - 3(C/\sigma_n)]\phi(C/\sigma_n)\}\ , \tag{6}$$

where $\phi(x)$ is the density function of the standard normal distribution $\Phi(x)$. (6) can be used to compute the P-value, and the α-percentile Z_α of Z may be approximated using the Cornish-Fisher approach as

$$Z_\alpha = \sigma_n \frac{Z_\alpha}{\sigma_n} \doteq \sigma_n\ [U_\alpha + \frac{1}{24}(U_\alpha^3 - 3U_\alpha)\ \gamma_{2,n}]\ , \tag{7}$$

where $U_\alpha = \Phi^{-1}(\alpha)$.

3. THE POWER OF THE TEST

A Monte Carlo experiment was conducted with a view to comparing the power of the Z-test, i.e., the test proposed in Section 2, with the powers

of the other well known tests. In this experiment, 1000 samples each of size $n = 20$ were obtained by simulation from the following distributions: (i) Uniform distribution, (ii) Cauchy distribution, (iii) Exponential distribution, (iv) Gamma (2) distribution, and (v) Beta (2,1) distribution. Again, the Marsaglia's random number package was used as a basis for generating the observations. The Z-test at 5% level of significance was performed on each of the samples in order to estimate its power at these alternatives. The power values of the Z-test, along with the corresponding values for competing tests, are presented in Table 1. These tests are separated into two groups. The first consists of tests designed specifically for testing the composite hypothesis of normality and the other of the well known EDF tests adapted for this purpose. Some of the values of the power functions are taken from Vasicek (1976), some others from Filliben (1975), and the remaining are estimated by us on the basis 1000 samples.

From the table it may be concluded that: (i) the Z-test is significantly superior to all other tests in the table at Beta (2,1) and Gamma (2) alternatives, (ii) the Z-test is superior or comparable with other tests in detecting the exponential alternative, (iii) at the Cauchy alternative the Z-test is inferior to all except Vasicek's k_3, Hartley and Pfaffenberger's S^2 and McDonald and Katti's L tests, and (iv) at the uniform alternative, where most tests have low power, the Z-test is poor.

TABLE 1 Empirical Powers of tests for normality against some alternatives ($n = 20$, $\alpha = 5\%$)

		Exponential	Gamma(2)	Beta(2,1)	Uniform	Cauchy
skewness β_1		2	1.414	-0.566	0	0
kurtosis β_2		9	6	2.4	1.8	---
	Z	.84	.55	.52	.05	.70
	$K_{3,20}$	.85	.45	.43	.44	.75
	L	.84	.31	.31	.15	.75
(I)	R	.82	.48	.20	.04	.92
	S^2	.29	.11	.15	.09	.34
	W	.84	.50	.35	.23	.88
	W'	.82	.48	.18	.04	.91
	K	.59	.33	.17	.12	.86
	W^2	.74	.45	.23	.16	.88
(II)	V	.71	.33	.20	.17	.87
	U^2	.70	.37	.23	.18	.88
	A^2	.82	.48	.28	.21	.98

$K_{3,20}$: Vasicek
L : McDonald-Katti
R : Filliben
S^2: Hartley-Pfaffenberger
W : Shapiro-Wilk
W': Shapiro-Francia
K : Kolmogorov-Smirnov
W^2: Cramér-von Mises
V : Kuiper
U : Watson
A^2 : Anderson-Darling

4. JOINT ASSESSMENT OF NORMALITY - AN ILLUSTRATION

As indicated in the introduction, the Z-test is well-suited to joint assessment of normality of several independent samples. We illustrate this point by considering the example discussed in Biometrika Tables for Statisticians (Pearson and Hartley (1972), p. 39) where a similar analysis is

done using the Shapiro-Wilk's W-test. The joint assessment involves obtaining the P-values of a test of normality applied to the independent samples and combining these P-values using a procedure such as Fisher's (1932). The P-value associated with a W-test is approximated from a Johnson - S_B distribution fit. The P-value associated with the Z-test, however, is available more readily from

$$\text{P-value} = 2 - 2\Phi(|Z|/\sigma_n) \tag{8}$$

or more accurately from (6). If $P_1, P_2, \ldots, P_k$ are the k P-values, then $T = -2 \sum_{i=1}^{k} \log P_i$ is distributed as a χ^2_{2k}-variable if the parent population is normal. We reject the normality hypothesis if T is large.

Example We take the first $k = 10$ samples of size $n = 5$, as in Biometrika Tables for Statisticians, Vol. 2, P. 39 for illustrating the joint assessment of normality using Shapiro-Wilk test, and find that the values of the Z-statistic and the corresponding P-values are as follows:

Table 2

Sample	1	2	3	4	5
Z_i	1.11067	2.08996	2.5950	1.13817	.034096
P_i	.271534	.035155	.007936	.259673	.973266

Sample	6	7	8	9	10
Z_i	.699262	1.44872	.776178	.051456	1.08966
P_i	.490654	.149498	.443918	.959664	.280701

The value of Fisher's combination statistic T_F for these is

$$T_F = -2 \sum_{1}^{10} \log P_i = 31.2$$

which is very close to the 95th percentile $\chi^2_{.95}(20) = 31.4$ of the chi-square distribution, indicating near-significance at the 5% level. The significance probability of these data based upon Shapiro-Wilk test is about 10%.

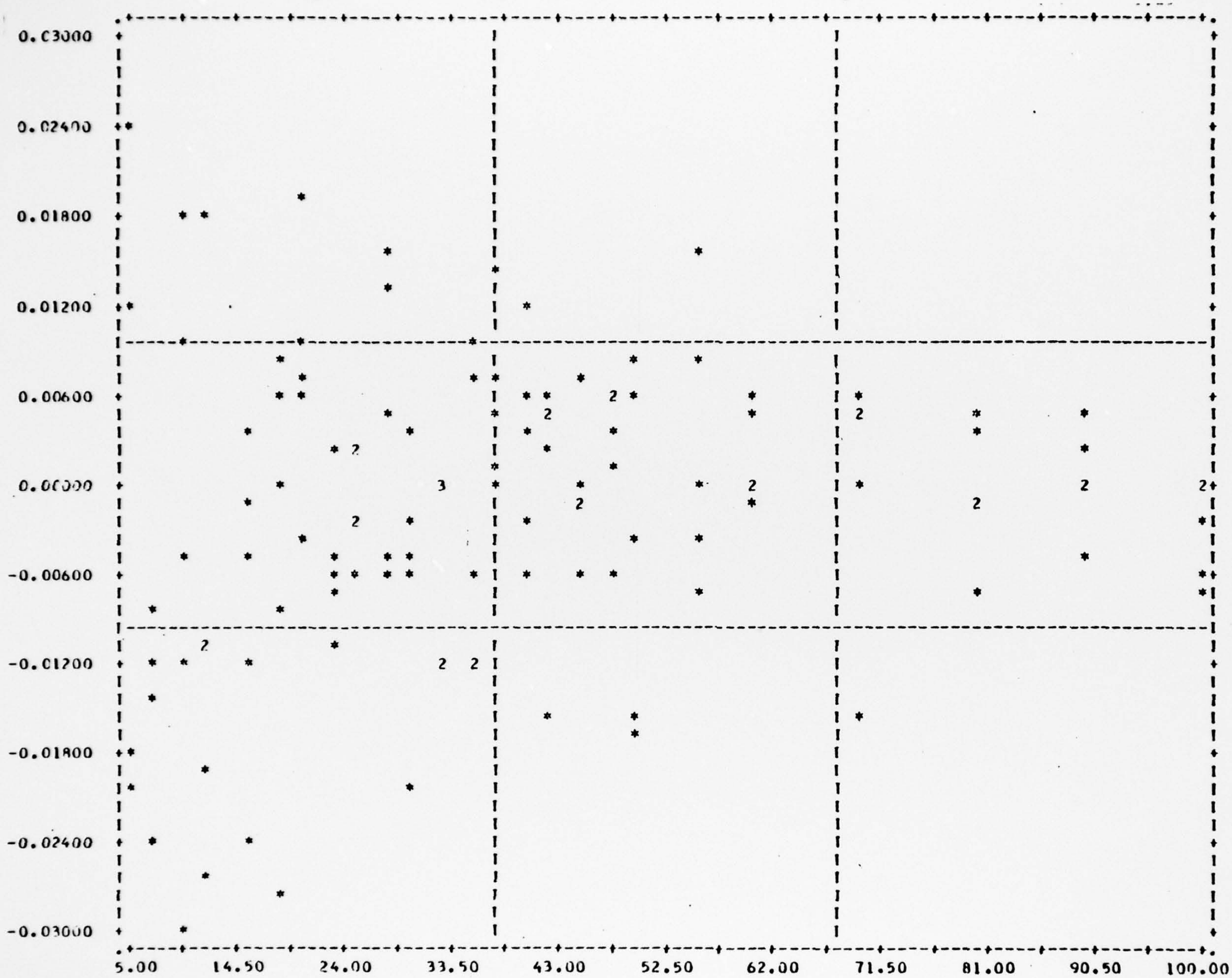
5. REMARKS

1. As mentioned in 2.1, there are many alternatives for obtaining the replications of $(\overline{X}, S^2)$. One example is to take all $\binom{n}{2}$ possible pairs of a sample $X_1, X_2, \ldots, X_n$ and compute the mean and variance from each pair as is done in McDonald and Katti (1975). However, the computation is more tedious than that of the procedure in 2.1. Moreover, our studies indicate that the use of the independence characterization with this method leads to a test very similar to the Z-test in power.

2. Among ~~all~~ the current tests for independence of a bivariate population, only Hoeffding's D statistic (1948) or its asymptotic equivalent B statistic due to Blum, Kiefer, and Rosenblatt (1961), is consistent. As might be expected, the powers of these tests are poor. A simple alternative to the product moment correlation as a measure for the dependence is the Spearman rank correlation.

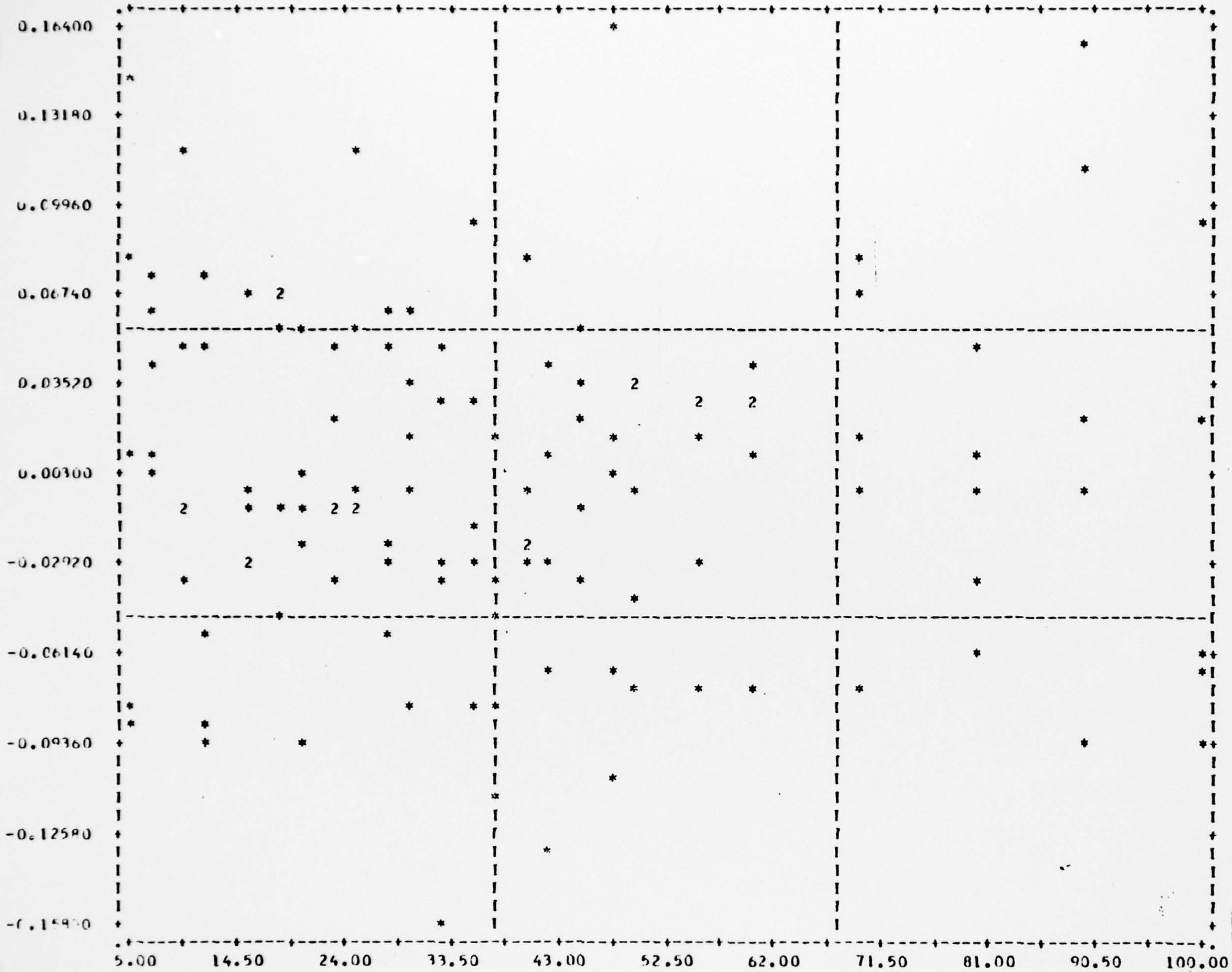
3. Remark 2 indicates the main limitation of Z-test, that it is not consistent against all alternatives. However, in small samples it seems to fare reasonably well with respect to other tests in power. It seems that Shapiro-Wilk test is still the most powerful omnibus test for normality. The Z-test introduced in this paper provides a simple alternative especially if joint assessment of normality is the consideration.

Figure 1 Scattergram of Mean (down) versus Sample Size N (across)



* Each point is based upon 1000 values.

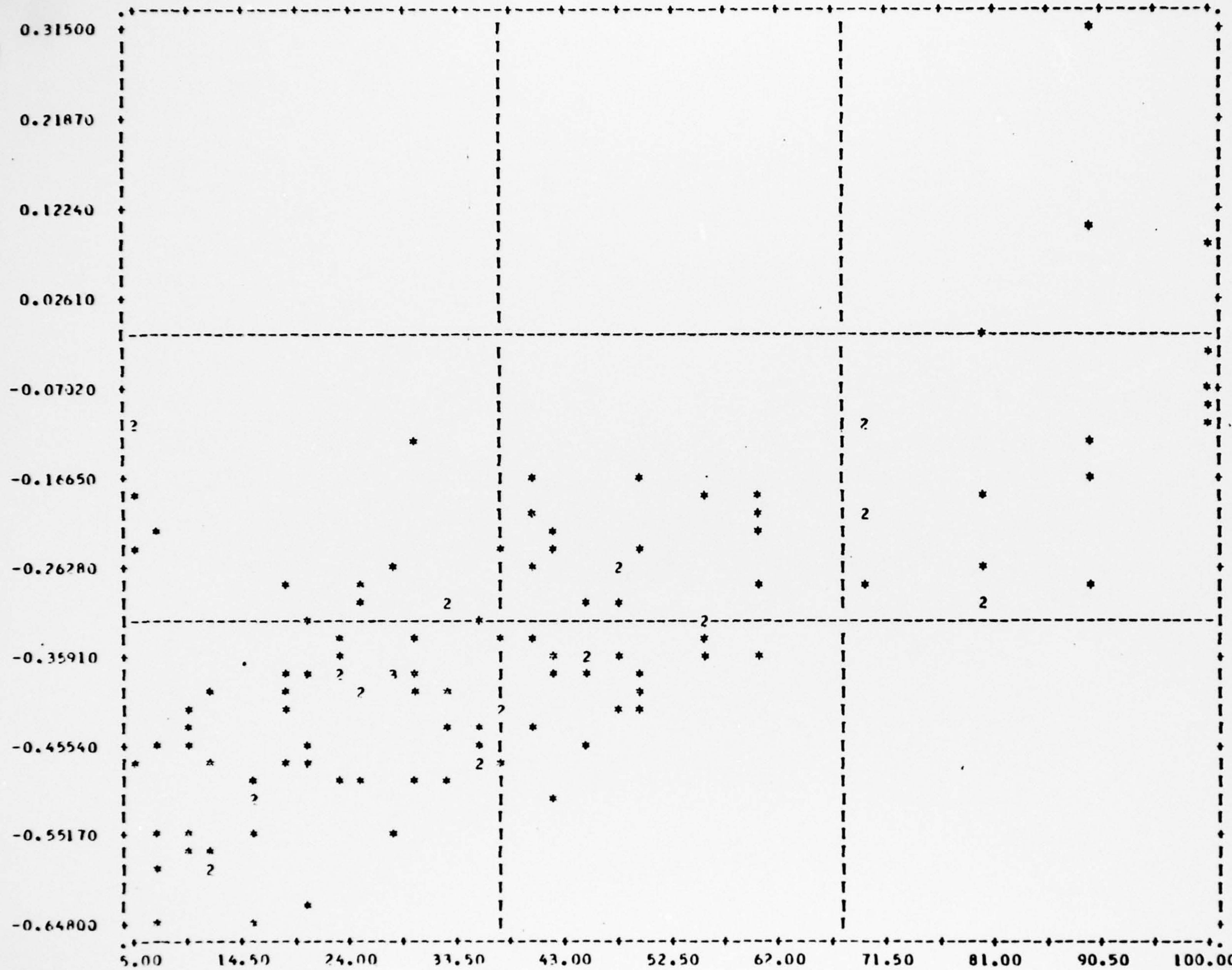
Figure 2 Scattergram of Skewness (down) versus Sample Size N (across)



* Each point is based upon 1000 values.

Figure 3 Scattergram of Excess of Kurtosis (down) versus

Sample Size N (across)



* Each point is based upon 1000 values.

APPENDIX:FORTRAN ROUTINE FOR COMPUTING
THE P-VALUE OF Z-STATISTIC

```
      SUBROUTINE ZTEST(X,N,U,V,Z,P)
C
C***********************************************************************
C     THIS SUBROUTINE COMPUTES THE P-VALUE OF Z-TEST FOR NORMALITY
C     USAGE    CALL ZTEST(X,N,Z,P)
C     X        INPUT VECTOR OF LENGTH N CONTAINING THE SAMPLE
C              OBSERVATIONS  .
C     N        NUMBER OF SAMPLE OBSERVATIONS
C     U        WORK AREA OF LENGTH N
C     V        WORK AREA OF LENGTH N
C     Z        SAMPLE VALUE OF Z-STATISTIC
C     P        P-VALUE OF Z-STATISTIC
C     REQUIRED ROUTINES ∈ FUNCTIONS
C     SUM(X,N)           FUNCTION SUBPROGRAM TO GIVE THE SUM OF X
C     SUMSQ(X,N)         FUNCTION SUBPROGRAM TO GIVE THE SUM SQUARE OF X
C     CROSPR(X,Y,N)      FUNCTION SUBPROGRAM TO GIVE THE SUM OF PRODUCT
C                        OF X AND Y
C     NORMAL(X,Y,P)      SUBROUTINE TO GIVE THE ORDINATE Y AND THE
C                        PROBABILITY P OF THE STANDARD NORMAL DISTRIBUTION
C                        AT X AND MAY BE SUBSTITUTED BY COMPARABLE ROUTINE
C***********************************************************************
C
      DIMENSION X(N),U(N),V(N)
      S=SUM(X,N)
      SSQ=SUMSQ(X,N)
      DO 10 I=1,N
      U(I)=S-X(I)
10    V(I)=(SSQ-X(I)*X(I)-U(I)*U(I)/(N-1))**(1./3.)
      CORR=(CROSPR(U,V,N)-SUM(U,N)*SUM(V,N)/N)/
     1((SUMSQ(U,N)-SUM(U,N)**2/N)*(SUMSQ(V,N)-SUM(V,N)**2/N))
     2**.5
      Z=0.5*ALOG((1+CORR)/(1-CORR))
      SIGMA=1./(0.591730+0.143559*N-0.002235*N*N+0.000016*N*N*N)
      GAMMA2=-11.697157/N+55.059097/(N*N)
      ZN=ABS(Z/SIGMA)
      CALL NORMAL(ZN,Y,P)
      P=2.-2.*(P-GAMMA2*(ZN*ZN*ZN-3*ZN)*Y/24.)
      RETURN
      END
C
C
      FUNCTION SUM(X,N)
      DIMENSION X(N)
      SUM=0.
      DO 10 I=1,N
10    SUM=SUM+X(I)
      RETURN
      END
C
      FUNCTION SUMSQ(X,N)
      DIMENSION X(N)
      SUMSQ=0.
      DO 10 I=1,N
10    SUMSQ=SUMSQ+X(I)*X(I)
      RETURN
      END
```

```
      FUNCTION CROSPR(X,Y,N)
      DIMENSION X(N),Y(N)
      CROSPR=0.
      DO 10 I=1,N
10    CROSPR=CROSPR+X(I)*Y(I)
      RETURN
      END
C
      SUBROUTINE NORMAL(X,Y,P)
      G=1.1283792*EXP(-(X*X/2.))
      Y=G/2.82842712
      XA=ABS(X)
      IF(XA.LT.2.5) GO TO 2
      U=1./(XA+1./(XA+2./(XA+3./(XA+4./(XA+5./(XA+6./(XA+7./(XA+8./(XA+9
     1./(XA+10./(XA+11./(XA+12./XA)))))))))))
      IF(X.GE.0) GO TO 1
      P=U*Y
      GO TO 4
1     P=1.-U*Y
      GO TO 4
2     ET=1.4142136/(1.4142136+0.3275911*XA)
      U=G*((((0.94064607*ET-1.28782245)*ET+1.2596513)*ET-0.252128668)*
     1ET+0.225836846)*ET
      IF(X.GE.0) GO TO 3
      P=U/2.
      GO TO 4
3     P=1.-U/2.
4     RETURN
      END
```

APL FUNCTION FOR COMPUTING THE P-VALUE OF Z-STATISTIC

X:	INPUT VECTOR OF SAMPLE OBSERVATIONS
NORM Z:	APL PUBLIC LIBRARY FUNCTION TO GIVE THE PROBABILITY OF STANDARD NORMAL DISTRIBUTION AT Z

```
∇ P←ZTEST X
[1] V←(((+/X*2)-(X*2))-(U*2)÷N-1)*(1÷3)∘U←(+/X)-X∘N←⍴X
[2] R←((+/U×V)-(+/U)×(+/V)÷N)÷(((+/U*2)-((+/U)*2)÷N)×(+/V*2)-((+/V)*2)÷N)*.5
[3] Z←|(0.5×⍟(1+R)÷1-R)×SIGMA←0.591730+(0.143559×N)+(¯0.002235×N*2)+0.000016×N*3
[4] GAMMA2←(¯11.697157÷N)+55.059097÷N*2
[5] P←2-2×((NORM Z)-(1÷24)×GAMMA2×((Z*3)-3×Z)×(2.718281828*(-(Z*2)÷2))÷(○2)*.5)
∇
```

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The mean and the variance of a random sample are indepently distributed if and only if the parent population is normal. This characterization is used as a basis for developing a test termed Z-test for the composite hypothesis of normality. The simplicity of Z-test results from the computational ease and the clean form of the finite sample null distribution of the test statistic Z. It compares reasonably in power with several well known tests of normality and is particularly suitable for a joint assessment of normality. A routine for computing the statistic and its P-value is given.

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